Simple and Compound interest calculations:

SIMPLE INTEREST:

Simple interest is given by,

S.I = P*n*R/100

Where in,

P- Principle in Rs

n- Number of years.

R- Rate of interest, expressed as %.

Let us arbitrarily take the value of "n" , the number of years to be 5.

Now,

S.I = 5*P*R/100 in which at any given year interest will be P*R/100.

COMPOUND INTEREST:

Here again, let us assume a scenario of 5 years.

1st YEAR:

Let us assume the initial principal to be P_1 .

If the rate of interest is R, Simple interest for 1st year is given by,

$S.I_1=(P_1*R)/100$ ……..(1)

2nd YEAR:

Principal for 2nd year is,

$P_2= P_1+S.I_1$

$= P_1 + ((P_1*R)/100)$ (From equation (1))

$P_2= P_1(1+ (R/100))= A_1$ (Where A_1 is amount after 1 year)……..(2)

$S.I_2 = P_2*R/100$

$= P_1(R/100)*(1+(R/100))$(From equation (2))……..(3)

3rd YEAR:

Principal for 3rd year is,

$P_3 = P_2 + S.I_2$

= $P_1(1+(R/100))+(P_1(R/100)*(1+(R/100)))$(From equations (2) and (3))

$P_3= P_1(1+(R/100))^2 = A_2$(Where A_2 is amount after 2 years)……(4)

$S.I_3 = P_3*R/100$

$= P_1(1+(R/100))^2*R/100$(From equation (4))……(5)

4^{th} YEAR:

Principal for 4^{th} year is,

$P_4 = P_3+ S.I_3$

$= P_1(1+(R/100))^2+ (P_1(1+(R/100))^2*R/100$(From equations (4) and (5))

$P_4= P_1(1+(R/100))^3= A_3$(Where A_3 is amount after 3 years)……..(6)

$S.I_4 = P_4*R/100$

$= P_1(1+(R/100))^3*R/100$(From equation (6))……(7)

5^{th} YEAR:

Principal for 5^{th} year is,

$P_5 = P_4 + S.I_4$

$= P_1(1+(R/100))^3+(P_1(1+(R/100))^3*R/100$(From equations (6) and (7))

$P_5 = P_1(1+(R/100))^4=A_4$(Where A_4 is amount after 4 years)……..(8)

$S.I_5 = P_5*R/100$

$= P_1(1+(R/100))^4*R/100$ (From equation (8))……..(9)

Here amount after 5 years equals principal for 6^{th}year(although 6^{th} year in our case is imaginary).

That is,

$A_5 = P_6= P_1(1+(R/100))^5$ (From logic akin to previous sequence).

In general,

$P_n = A_{n-1} = P_1(1+(R/100))^{n-1}$ where ‘n’ denotes number of years……(10)

In general ,when compounded, interest for n^{th} year is given by,

$S.I_n = (P_1*R/100)*(1+(R/100))^{n-1}$……..(11)

Difference between interests for n^{th} year and $(n-1)^{th}$ year, when compounded,is given by,

$S.I_n - S.I_{(n-1)} = (P_1*R/100)*(1+(R/100))^{(n-1)} - ((P_1*R/100)*(1+(R/100))^{(n-2)}$

(From equation (11))

$= P_1*(R/100)^2*(1+(R/100))^{(n-2)}$……(12)

A case when the amount becomes 'x' times the principal in the simple interest mode:

When principal gets doubled (A=2*P),

P+(P*n*R/100)=2*P

n = 100/R

When principal becomes 1.5 times its initial value (A=1.5*P),

P + (P*n*R/100)=1.5*P

n = 50/R

In general, when principal becomes 'x' times its initial value (A=x*P),

n = (x-1)*100/R……(13)

Example : When the initial principal is Rs 1000 and the rate of interest is 10% compounded annually,calculate

1. The principal for 3rd year?
2. The interest for 4th year?
3. The difference in interest for 5th year and 4th year?

Solution:

1. Principal for 3rd year = $P_3 = 1000*(1+(10/100))^2$ (From equation (10))
   = Rs 1210.
2. Interest for 4th year = $S.I_4 = (1000*10/100)*(1+(10/100))^3$(From equation (11))
   = Rs 133.1.
3. Difference in interest for 5th and 4th year is,
   $= 1000*(10/100)^2 *(1+ (10/100))^3$ (From equation (12))
   = Rs 13.31.